**Electrocardiogram and potential difference**

**Introduction**

The heart pumps blood around the body at all times by contracting and relaxing continuously. The contraction is triggered whenever a wave of electrical impulses passes through the heart. As a first approximation, the heart can be modeled by a dipole moment vector that is changing in magnitude and direction during a heartbeat. The resulting electric field produced by this dipole generates a variable electric potential. The electrical potentials generated by the field of the dipole can be recorded by placing two electrodes on the skin on opposite sides of the heart. This is known as an electrocardiogram (ECG).

**1. Short questions**

|  |  |  |
| --- | --- | --- |
|  |  |  |
| Figure a. | Figure b. | Figure c. |

The figures show the orientation of the dipole moment vector representing the state of the heart at three instants in time during the heart cycle. Consider measuring two potential differences, and , where 1, 2, 3, and 4 are points around the dipole and indicated in the figures.

Circle one:

a) at

b) at

**Answer:**

a) at

b) at

Consider the **instant t=t1**, the equipotential lines are shown in the figure below:



We can see that points 3 and 4 are on an equipotential line, therefore , so

What happens with the electric potential difference ? We can do the detailed calculation or we can make the qualitative argument below.

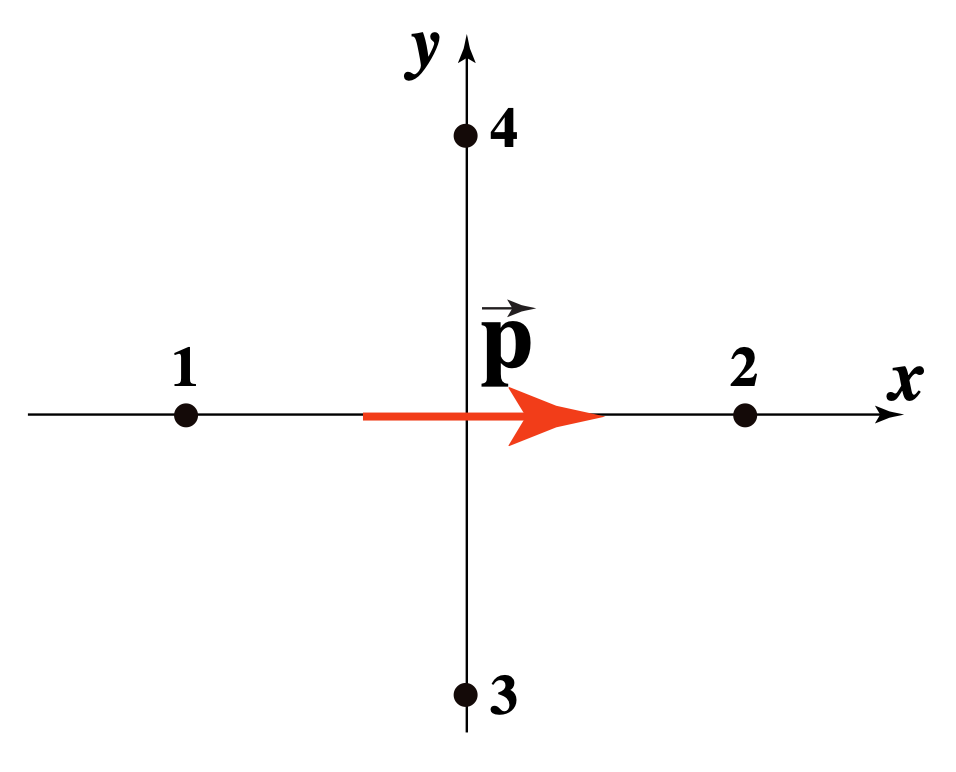
At point 2 the electric potential **is positive**. Why? The E-line where point 2 is located, leaves the + charge and goes to infinity, therefore the electric potential **decreases** with the distance away from the positive charge (the electric potential decreases along an E-line in the same direction of the field), so .

At point 1, the electric potential **is negative**. Why? The E-line where point 1 is located, starts at infinity and ends at the negative charge, therefore the electric potential **increases** with the distance away from the negative charge (the electric potential increases along an E-line in the opposite direction to the field), so

Therefore, if and then .

**At t = t2** we repeat the same reasoning. There is an equipotential line along the x-axis, so , but

**2. Problem**

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In the figure above, the dipole moment vector modeling the heart is shown at a given instant during the heartbeat cycle. For parts a) to d), assume that the dipole is composed by a positive and a negative charge of magnitude , centered at the origin and separated by a distance

a) Calculate and , the electric potential with reference at infinity at points 4 and 3 on the y-axis.

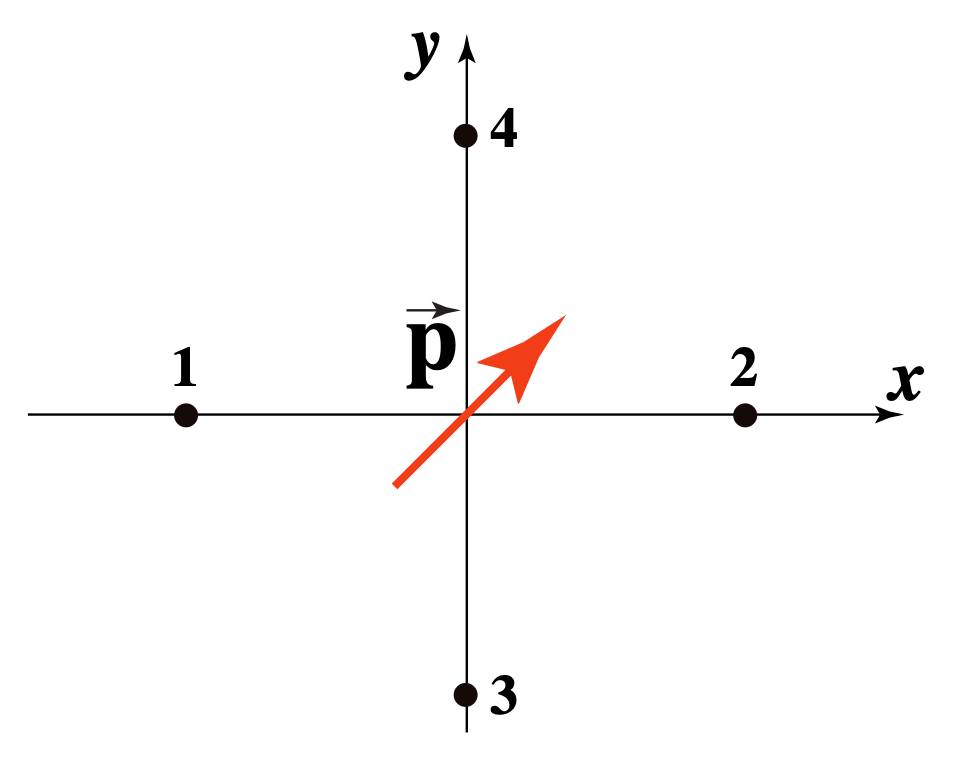
b) Calculate , the electric potential with reference at infinity at point 2 located on the x-axis at

c) Calculate , the electric potential with reference at infinity at point 1 located on the x-axis at

d) Calculate the electric potential difference between points 2 and 1, . Express your answer in terms of the magnitude of the dipole moment vector and as needed.

e) What is the expression of obtained in part d) for ?

f) Consider the instant in time at which the dipole moment vector is oriented as shown below:

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At that instant, the moment dipole vector has two components (one along the x and one along the y axis) and it can be written as:

Use the results of parts a) to d) to find an expression for and at the instant shown in the figure. Assume points 4 and 3 are are at positions is at position , and points 2 and 1 are at positions Express your answers in terms of . Assume that the electrodes are far enough from the center of the dipole so that >> a and >>a.

**Solution**

a) Calculate and , the electric potential with reference at infinity at points 4 and 3 on the y-axis.

**Answer**: To calculate the electric potential at any point P in space we use the sum of the electric potential of the two point charges, for the +q and for the -q at that point:

where is the position of point P, and are the positions of the +q and -q, respectively.

For points along the y-axis:

|  |  |
| --- | --- |
| **Point 4:** | |
| Electric potential produced by +q: | Electric potential produced by -q: |
|  |  |
|  | |

We will obtain the same answer for point 3. So, at any point along the y-axis V = 0, therefore , so as expected because the y-axis is an equipotential line.

b)Calculate , the electric potential with reference at infinity at point 2 located on the x-axis at

**Answer: For points along the x-axis, Point 2**

|  |  |
| --- | --- |
| **Point 2:** | |
| Electric potential produced by +q: | Electric potential produced by -q: |
|  |  |
|  | |

c) Calculate , the electric potential with reference at infinity at point 1 located on the x-axis at

**Answer: for** **Point 1**

|  |  |
| --- | --- |
| **Point 1:** | |
| Electric potential produced by +q: | Electric potential produced by -q: |
|  |  |
|  | |

d) Calculate the electric potential difference between points 2 and 1, . Express your answer in terms of the magnitude of the dipole moment vector and as needed.

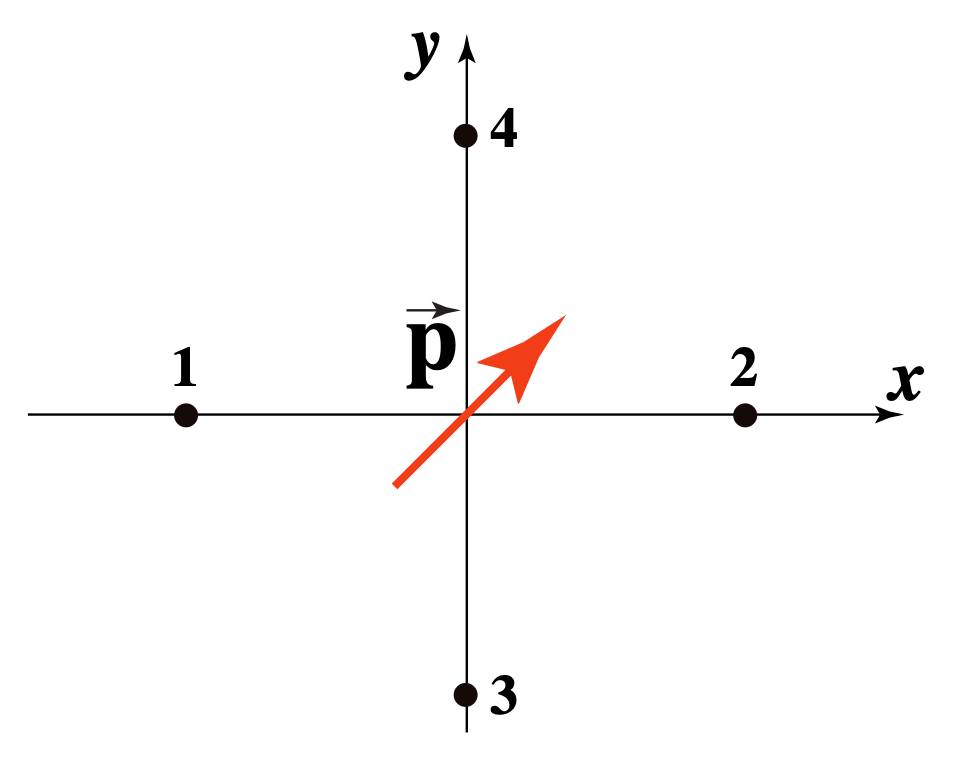
**Answer:** Use the results from parts c) and d)

Using that , the expression above becomes:

e) What is the expression of obtained in part d) for ?

For

f) Consider the instant in time at which the dipole moment vector is oriented as shown below:

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At that instant, the moment dipole vector has two components (one along the x and one along the y axis) and it can be written as:

Use the results of parts a) to d) to find an expression for and at the instant shown in the figure. Assume points 4 and 3 are are at positions is at position , and points 2 and 1 are at positions Express your answers in terms of . Assume that the electrodes are far enough from the center of the dipole so that >> a and >>a.

f) Putting things together, the electrodes 1 and 2 will measure the x-component of the dipole, whereas electrodes 3 and 4 the y – component of the moment dipole vector.

Far from the origin of the coordinate system >> a,

If the electrodes are far enough so that , and. >> a , then we can say

We see that at distance larger that a, the electric potential at points on the x and y axis behaves as and .

If we use the definition of change of electric potential, we can calculate the E-field knowing V:

This result is consistent with what we showed in the in-class problem 7-a and 7-b , week 1, far from the dipole the E-field depends on the distance as .

**References:**

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